4. Material Balance Calculations for Oil Reservoirs

A general material balance equation that can be applied to all reservoir types was first developed by Schilthuis in 1936. Although it is a tank model equation, it can provide great insight for the practicing reservoir engineer. It is written from start of production to any time (t) as follows:

Expansion of oil in the oil zone +
Expansion of gas in the gas zone +
Expansion of connate water in the oil and gas zones +
Contraction of pore volume in the oil and gas zones +
Water influx + Water injected + Gas injected =
Oil produced + Gas produced + Water produced

Mathematically, this can be written as:

\[ N(B_t - B_n) + G(B_g - B_g) + \left( NB_n + GB_g \right) \left( \frac{C_w S_{wi}}{I - S_{wi}} \right) \Delta p_t + \left( NB_n + GB_g \right) \left( \frac{C_f}{I - S_{wi}} \right) \Delta p_t + W_e + W_i B_{ho} + G_i B_g = N_p B_i + N_p \left( R_p - R_{soi} \right) B_g + W_p B_w \]

Where:

- \( N \) = initial oil in place, STB
- \( N_p \) = cumulative oil produced, STB
- \( G \) = initial gas in place, SCF
- \( G_i \) = cumulative gas injected into reservoir, SCF
- \( G_p \) = cumulative gas produced, SCF
- \( W_e \) = water influx into reservoir, bbl
- \( W_i \) = cumulative water injected into reservoir, STB
- \( W_p \) = cumulative water produced, STB
- \( B_{ni} \) = initial two-phase formation volume factor, bbl/STB = \( B_{oi} \)
- \( B_{oi} \) = initial oil formation volume factor, bbl/STB
- \( B_{gi} \) = initial gas formation volume factor, bbl/SCF
- \( B_t \) = two-phase formation volume factor, bbl/STB = \( B_{oi} + (R_{soi} - R_{so}) \) \( B_g \)
- \( B_{oi} \) = oil formation volume factor, bbl/STB
- \( B_{gi} \) = gas formation volume factor, bbl/SCF
- \( B_{gi} \) = water formation volume factor, bbl/STB
- \( B_{gi} \) = injected gas formation volume factor, bbl/SCF
- \( B_{iw} \) = injected water formation volume factor, bbl/STB
- \( R_{soi} \) = initial solution gas-oil ratio, SCF/STB
- \( R_{so} \) = solution gas-oil ratio, SCF/STB
- \( R_p \) = cumulative produced gas-oil ratio, SCF/STB
- \( C_f \) = formation compressibility, psia\(^{-1}\)
- \( C_w \) = water isothermal compressibility, psia\(^{-1}\)
- \( S_{wi} \) = initial water saturation
- \( \Delta p_t \) = reservoir pressure drop, psia = \( p_i - p(t) \)
- \( p(t) \) = current reservoir pressure, psia
4.1. The MBE as a Straight Line

Normally, when using the material balance equation, each pressure and the corresponding production data is considered as being a separate point from other pressure values. From each separate point, a calculation is made and the results of these calculations are averaged. However, a method is required to make use of all data points with the requirement that these points must yield solutions to the material balance equation that behave linearly to obtain values of the independent variable. The straight-line method begins with the material balance written as:

\[
N(B_i - B_{ni}) + G(B_s - B_{gs}) + (NB_{ni} + GB_{gi}) \left( \frac{C_f + C_w S_{wi}}{1 - S_{wi}} \right) \Delta p_i \\
+ W_e + W_i B_{iw} + G_1 B_{lg} \\
= N_p B_i + N_p \left( R_p - R_{stoi} \right) B_g + W_p B_w
\]

Defining the ratio of the initial gas cap volume to the initial oil volume as:

\[
m = \frac{\text{initial gas cap volume}}{\text{initial oil volume}} = \frac{GB_{gi}}{NB_{ni}}
\]

and plugging into the equation yields:

\[
N(B_i - B_{ni}) + Nm \frac{B_{ni}}{B_{gi}} \left( B_g - B_{gs} \right) + (NB_{ni} + Nm B_{ni}) \left( \frac{C_f + C_w S_{wi}}{1 - S_{wi}} \right) \Delta p_i \\
+ W_e + W_i B_{iw} + G_1 B_{lg} \\
= N_p B_i + N_p \left( R_p - R_{stoi} \right) B_g + W_p B_w
\]

Let:

\[
E_o = B_i - B_{ni}
\]

\[
E_g = \frac{B_{ni}}{B_{gi}} \left( B_g - B_{gs} \right)
\]

\[
E_{f,w} = (1 + m)B_{ni} \left( \frac{C_f + C_w S_{wi}}{1 - S_{wi}} \right) \Delta p_i
\]

\[
F = N_p \left[ B_i + \left( R_p - R_{stoi} \right) B_g \right] + W_p B_w - W_i B_{iw} - G_1 B_{lg}
\]

Thus we obtain:

\[
F = NE_o + mNE_g + NE_{f,w} + W_e
\]

\[
= N \left( E_o + mE_g + E_{f,w} \right) + W_e
\]
The following cases are considered:

1. No gas cap, negligible compressibilities, and no water influx

\[ F = NE_o \]

2. Negligible compressibilities, and no water influx

\[ F = NE_o + NmE_g \]

\[ \frac{F}{E_o} = N + Nm \frac{E_g}{E_o} \]

Which is written as \( y = b + x \). This would suggest that a plot of \( F/E_o \) as the y coordinate versus \( E_g/E_o \) as the x coordinate would yield a straight line with slope equal to \( mN \) and intercept equal to \( N \).

3. Including compressibilities and water influx, let:

\[ D = E_o + mE_g + E_{f,w} \]

Dividing through by \( D \), we get:

\[ \frac{F}{D} = N + \frac{W_e}{D} \]

Which is written as \( y = b + x \). This would suggest that a plot of \( F/D \) as the y coordinate and \( W_e/D \) as the x coordinate would yield a straight line with slope equal to 1 and intercept equal to \( N \).

### 4.2 Drive Indexes from the MBE

The three major driving mechanisms are:

1. Depletion drive (oil zone oil expansion),
2. Segregation drive (gas zone gas expansion), and
3. Water drive (water zone water influx).

To determine the relative magnitude of each of these driving mechanisms, the compressibility term in the material balance equation is neglected and the equation is rearranged as follows:

\[ N \left( B_i - B_o \right) + G \left( B_g - B_{gi} \right) + \left( W_e - W_p B_w \right) = N_p \left[ B_i + (R_p - R_{soi}) B_g \right] \]

Dividing through by the right hand side of the equation yields:

\[ \frac{N \left( B_i - B_o \right)}{N_p \left[ B_i + (R_p - R_{soi}) B_g \right]} + \frac{G \left( B_g - B_{gi} \right)}{N_p \left[ B_i + (R_p - R_{soi}) B_g \right]} + \frac{\left( W_e - W_p B_w \right)}{N_p \left[ B_i + (R_p - R_{soi}) B_g \right]} = 1 \]
The terms on the left hand side of equation (3) represent the depletion drive index (DDI), the segregation drive index (SDI), and the water drive index (WDI) respectively. Thus, using Pirson's abbreviations, we write:

$$\text{DDI} + \text{SDI} + \text{WDI} = 1$$

The following examples should clarify the errors that creep in during the calculations of oil and gas reserves.

**Example #5: Given the following data for an oil field**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of bulk oil zone</td>
<td>112,000 acre-ft</td>
</tr>
<tr>
<td>Volume of bulk gas zone</td>
<td>19,600 acre-ft</td>
</tr>
<tr>
<td>Initial reservoir pressure</td>
<td>2710 psia</td>
</tr>
<tr>
<td>Initial oil FVF</td>
<td>1.340 bbl/STB</td>
</tr>
<tr>
<td>Initial gas FVF</td>
<td>0.006266 ft$^3$/SCF</td>
</tr>
<tr>
<td>Initial dissolved GOR</td>
<td>562 SCF/STB</td>
</tr>
<tr>
<td>Oil produced during the interval</td>
<td>20 MM STB</td>
</tr>
<tr>
<td>Reservoir pressure at the end of the interval</td>
<td>2000 psia</td>
</tr>
<tr>
<td>Average produced GOR</td>
<td>700 SCF/STB</td>
</tr>
<tr>
<td>Two-phase FVF at 2000 psia</td>
<td>1.4954 bbl/STB</td>
</tr>
<tr>
<td>Volume of water encroached</td>
<td>11.58 MM bbl</td>
</tr>
<tr>
<td>Volume of water produced</td>
<td>1.05 MM STB</td>
</tr>
<tr>
<td>Water FVF</td>
<td>1.028 bbl/STB</td>
</tr>
<tr>
<td>Gas FVF at 2000 psia</td>
<td>0.008479 ft$^3$/SCF</td>
</tr>
</tbody>
</table>

The following values will be calculated:

1. The stock tank oil initially in place.
2. The driving indexes.
3. Discussion of results.

**Solution:**

1. The material balance equation is written as:

$$N(B_i - B_n) + G(B_g - B_{gi}) = N_p \left[ B_i + (R_p - R_{io}) B_g \right] (W_e - W_p B_w)$$

Define the ratio of the initial gas cap volume to the initial oil volume as:

$$m = \frac{GB_{gi}}{NB_n}$$

we get:

$$N(B_i - B_n) + Nm \frac{B_n}{B_{gi}} (B_g - B_{gi}) = N_p \left[ B_i + (R_p - R_{io}) B_g \right] (W_e - W_p B_w)$$

and solve for $N$, we get:
\[ N = \frac{N_p \left[ B_i + \left( R_p - R_{soi} \right) B_g \right] - \left( W_e - W_p B_w \right)}{(B_i - B_n) + m \frac{B_w}{B_g} (B_g - B_{gi})} \]

Since:

\[
\begin{align*}
N_p &= 20 \times 10^6 \text{ STB} \\
B_i &= 1.4954 \text{ bbl/STB} \\
R_p &= 700 \text{ SCF/STB} \\
R_{soi} &= 562 \text{ SCF/STB} \\
B_g &= 0.008479 \text{ ft}^3/\text{SCF} = 0.008479/5.6146 = 0.001510 \text{ bbl/SCF} \\
W_e &= 11.58 \times 10^6 \text{ bbl} \\
W_p &= 1.05 \times 10^6 \text{ STB} \\
B_w &= 1.028 \text{ bbl/STB} \\
B_{ti} &= 1.34 \text{ bbl/STB} \\
m &= GB_g/\text{NB}_t = 19.600/112.000 = 0.175 \\
B_{gi} &= 0.006266 \text{ ft}^3/\text{SCF} = 0.006266/5.6146 = 0.001116 \text{ bbl/SCF} \\
\end{align*}
\]

Thus:

\[
N = \frac{20 \left[ 1.4954 + (700 - 562) \times 0.001510 \right] - (11.58 \times 1.05 \times 1.028)}{(1.4954 - 1.34) + 0.175(0.001510 - 0.001116)} \times 10^6
\]

\[
= 98.97 \text{ MM STB}
\]

2. In terms of drive indexes, the material balance equation is written as:

\[
N \left( B_i - B_n \right) \left( B_i + (R_p - R_{soi}) B_g \right) + G \left( B_g - B_{gi} \right) + \left( W_e - W_p B_w \right) = I
\]

Thus the depletion drive index (DDI) is given by:

\[
\frac{N\left( B_i - B_n \right)}{N_p \left( B_i + (R_p - R_{soi}) B_g \right)} = \frac{98.97 \times 10^6 \times (1.4954 - 1.34)}{20 \times 10^6 \times (1.4954 + (700 - 562) \times 0.001510)} = 0.45
\]

The segregation drive index (SDI) is given by:

\[
\frac{N \times m B_w (B_g - B_{gi})}{B_{gi} \left( B_i + (R_p - R_{soi}) B_g \right)} = \frac{98.97 \times 10^6 \times 0.175 \times (1.34 \times 0.001510 - 0.001116)}{20 \times 10^6 \times (1.4954 + (700 - 562) \times 0.001510)} = 0.24
\]
The water drive index (WDI) is given by:

\[
\frac{(W_e - W_p B_e)}{N_p B_t + (R_p - R_w) B_s} = \frac{(11.58 \times 10^6 - 1.05 \times 1.028 \times 10^6)}{20 \times 10^6 [1.4954 + (700 - 562)0.001510]} = 0.31
\]

3. The drive mechanisms as calculated in part (2) indicate that when the reservoir pressure has declined from 2710 psia to 2000 psia, 45% of the total production was by oil expansion, 31% was by water drive, and 24% was by gas cap expansion.

This concludes the solution.

**Example #6: Given the following data for an oil field**

A gas cap reservoir is estimated, from volumetric calculations, to have an initial oil volume \( N \) of \( 115 \times 10^6 \) STB. The cumulative oil production \( N_p \) and cumulative gas oil ratio \( R_p \) are listed in the following table as functions of the average reservoir pressure over the first few years of production. Assume that \( p_i = p_b = 3330 \) psia. The size of the gas cap is uncertain with the best estimate, based on geological information, giving the value of \( m = 0.4 \). Is this figure confirmed by the production and pressure history? If not, what is the correct value of \( m \)?

<table>
<thead>
<tr>
<th>Pressure psia</th>
<th>( N_p ) MM STB</th>
<th>( R_p ) SCF/STB</th>
<th>( B_o ) BBL/STB</th>
<th>( R_{so} ) SCF/STB</th>
<th>( B_g ) bbl/SCF</th>
</tr>
</thead>
<tbody>
<tr>
<td>3330</td>
<td>0</td>
<td>0</td>
<td>1.2511</td>
<td>510</td>
<td>0.00087</td>
</tr>
<tr>
<td>3150</td>
<td>3.295</td>
<td>1050</td>
<td>1.2353</td>
<td>477</td>
<td>0.00092</td>
</tr>
<tr>
<td>3000</td>
<td>5.903</td>
<td>1060</td>
<td>1.2222</td>
<td>450</td>
<td>0.00096</td>
</tr>
<tr>
<td>2850</td>
<td>8.852</td>
<td>1160</td>
<td>1.2122</td>
<td>425</td>
<td>0.00101</td>
</tr>
<tr>
<td>2700</td>
<td>11.503</td>
<td>1235</td>
<td>1.2022</td>
<td>401</td>
<td>0.00107</td>
</tr>
<tr>
<td>2550</td>
<td>14.513</td>
<td>1265</td>
<td>1.1922</td>
<td>375</td>
<td>0.00113</td>
</tr>
<tr>
<td>2400</td>
<td>17.73</td>
<td>1300</td>
<td>1.1822</td>
<td>352</td>
<td>0.00120</td>
</tr>
</tbody>
</table>

**Solution:**

Calculate the parameters \( F, E_o, E_g \) as given by the above equations:

<table>
<thead>
<tr>
<th>( B_t ) BBL/STB</th>
<th>( F ) MM/RB</th>
<th>( E_o ) RB/STB</th>
<th>( E_g ) RB/SCF</th>
<th>( F/E_o ) MM/STB</th>
<th>( E_g/E_o )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2511</td>
<td>5.8073</td>
<td>0.014560</td>
<td>0.071902299</td>
<td>398.8534135</td>
<td>4.938344701</td>
</tr>
<tr>
<td>1.2798</td>
<td>10.6714</td>
<td>0.028700</td>
<td>0.129424138</td>
<td>371.8272962</td>
<td>4.509551844</td>
</tr>
<tr>
<td>1.29805</td>
<td>17.3017</td>
<td>0.046950</td>
<td>0.201326437</td>
<td>368.5128136</td>
<td>4.28810302</td>
</tr>
<tr>
<td>1.31883</td>
<td>24.0940</td>
<td>0.067730</td>
<td>0.287609195</td>
<td>355.7353276</td>
<td>4.246407728</td>
</tr>
<tr>
<td>1.34475</td>
<td>31.8981</td>
<td>0.093650</td>
<td>0.373891954</td>
<td>340.6099594</td>
<td>3.992439445</td>
</tr>
<tr>
<td>1.3718</td>
<td>41.1301</td>
<td>0.120700</td>
<td>0.474555172</td>
<td>340.7626678</td>
<td>3.931691569</td>
</tr>
</tbody>
</table>

The plot of \( F/E_o \) versus \( E_g/E_o \) is shown next:
Figure 6: F/Eo vs. Eg/Eo Plot

The best fit is expressed by:

\[
\frac{F}{E_o} = 108.7 + 58.83 \frac{E_g}{E_o}
\]

Therefore, \(N = 108.7\) MM STB and \(m = \frac{58.83}{108.7} = 0.54\).

This concludes the solution of this problem.

Example #7: Given the following data for an oil field

A gas cap reservoir is estimated, from volumetric calculations, to have an initial oil volume \(N\) of \(47 \times 10^6\) STB. The cumulative oil production \(N_p\) and cumulative gas oil ratio \(R_p\) are listed in the following table as functions of the average reservoir pressure over the first few years of production. Other pertinent data are also supplied. Assume \(p_i = p_b = 3640\) psia. The size of the gas cap is uncertain with the best estimate, based on geological information, giving the value of \(m = 0.0\). Is this figure confirmed by the production history? If not, what is the correct value of \(m\)?

\begin{align*}
\text{pi} & = 3640 & \text{psia} \\
\text{Cf} & = 0.000004 & \text{psia}\-1 \\
\text{Cw} & = 0.000003 & \text{psia}\-1 \\
\text{Swi} & = 0.25 & \\
\text{Bw} & = 1.025 & \text{psia} \\
\text{m} & = 0 & \\
\end{align*}
Solution:

Calculate the parameters F, E<sub>o</sub>, E<sub>g</sub>, E<sub>f,w</sub>, and D, as given by the above equations:

\[
\frac{F}{D} = 0.0071 + 48.067e^6 \frac{W_e}{D}
\]

Therefore, N = 48 MM STB and m = 0.0071.

This concludes the solution of this problem.
Figure 7: F/Eo vs. Eg/Eo Plot

$y = 0.0071x + 48.067$